NAME: __Solutions__

Test Prep 1

This test prep includes the two problems below. If you finish the front, try the problems on the back which are worth extra (if you miss a point on the front, I will give you the point back if you have a question correct on the back). If you get everything (front and back) correct, I'll give you 11 out of 10 for this test prep. Most likely you won't have time to do the back, which is fine, just make sure you finish the front.

You get 10 minutes!

1. Evaluate $\int x^3 \ln(x)\, dx$ (I encourage you to check your final answer by differentiating).

$$u = \ln(x) \qquad dv = x^3 dx$$
$$du = \frac{1}{x} dx \qquad v = \frac{1}{4}x^4$$

$$= \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^3 dx$$

$$= \boxed{\frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + C}$$

2. Find the numbers A, B, and C that make the following true for all valid values of x

$$\frac{5-x}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}.$$

$$5 - x = A(x^2+4) + (Bx+c)\,x$$

$$x = 0 \Rightarrow \quad 5 - 0 = A(4) + 0 \Rightarrow \boxed{A = \frac{5}{4}}$$

Thus,
$$5 - x = \frac{5}{4}(x^2+4) + (Bx+c)x$$
$$\Rightarrow 5 - x = \frac{5}{4}x^2 + 5 + Bx^2 + Cx$$

COMPARING COEFFICIENTS ON BOTH SIDES:

$$x^2: \quad 0 = \frac{5}{4} + B \Rightarrow \boxed{B = -\frac{5}{4}}$$
$$x^1: \quad -1 = C \qquad \Rightarrow \boxed{C = -1}$$
$$x^0: \quad 5 = 5 \checkmark$$

Extra Practice Problems:

1. $\int 7 + \sec(x)\tan(x) + \cos^2(5x)\,dx$

$$7x + \sec(x) + \int \frac{1}{2}(1 + \cos(10x))\,dx$$

$$7x + \sec(x) + \frac{1}{2}x + \frac{1}{20}\sin(10x) + C$$

$$\boxed{\frac{15}{2}x + \sec(x) + \frac{1}{20}\sin(10x) + C}$$

2. $\int \frac{x+2}{x^2+1}\,dx$

$$\int \frac{x}{x^2+1}\,dx + \int \frac{2}{x^2+1}\,dx$$

$$\int \frac{x}{x^2+1}\,dx + 2\tan^{-1}(x)$$

$u = x^2 + 1$
$du = 2x\,dx$
$dx = \frac{1}{2x}\,du$

$$\int \frac{x}{u}\frac{1}{2x}\,du + 2\tan^{-1}(x)$$

$$\frac{1}{2}\ln|u| + 2\tan^{-1}(x) + C$$

$$\boxed{\frac{1}{2}\ln(x^2+1) + 2\tan^{-1}(x) + C}$$